

# Controlled-dipole quantum memory

Adam Green<sup>1,\*</sup>, Yang Han<sup>1,2,\*</sup>, Khabat Heshami<sup>1,\*</sup>, Arnaud Rispé<sup>1</sup>, Erhan Saglamyurek<sup>1</sup>, Neil Sinclair<sup>1</sup>, Wolfgang Tittel<sup>1</sup>, and Christoph Simon<sup>1</sup>

<sup>1</sup> *Institute for Quantum Information Science and Department of Physics and Astronomy,  
University of Calgary, Calgary T2N 1N4, Alberta, Canada*

<sup>2</sup> *College of Science, National University of Defense Technology, Changsha 410073, People's Republic of China*

\* *These authors contributed equally to this work.*

(Dated: June 20, 2011)

We present a quantum memory protocol for photons that is based on the direct control of the transition dipole moment. We focus on the case where the light-matter interaction is enhanced by a cavity. We show that the optimal write process (maximizing the storage efficiency) is related to the optimal read process by a reversal of the *effective time*  $\tau = \int dt g^2(t)/\kappa$ , where  $g(t)$  is the time-dependent coupling and  $\kappa$  is the cavity decay rate. We discuss the implementation of the protocol in certain rare-earth ion doped crystals, where transitions can be turned on and off by switching a magnetic field.

Quantum memories for light are devices that allow one to store and retrieve light in a way that preserves its quantum state [1–3]. They are essential components for optical quantum information processing, notably for quantum repeaters [4]. All quantum memories require a way of switching the coupling between the light and the material system (which is used as the memory) on and off in a controlled way. In the case of memories based on electromagnetically induced transparency or off-resonant Raman transitions [1, 5–8] the coupling is controlled by a laser beam, which is typically much more intense than the signal that one aims to store. In contrast, in the case of photon-echo based memories [3, 9, 10] the coupling is controlled in a more indirect way via the dephasing of the atoms in the storage medium. This typically requires spectral tailoring of the medium by optical pumping before the signal can be stored.

Here we consider a way of controlling the light-matter interaction that is different from the mentioned examples, and that is particularly simple from a conceptual point of view, namely the direct control of the transition dipole element of the relevant optical transition. This is motivated by recent demonstrations that transition dipoles can be turned on and off in certain solid-state systems, in particular in rare-earth doped crystals by applying magnetic fields [11, 12], and for NV centers in diamond by ap-

plying electric fields [13]. We consider the case where the storage medium is placed inside an optical cavity [14–16]. This both enhances the light-matter interaction, which is desirable for achieving high efficiencies, and simplifies the equations of motion, thus clearly bringing out the basic principles of the memory dynamics.

We consider an ensemble of two-level atoms coupled to a cavity mode, see Fig. 1. We ignore the spatial dependence of the light-matter interaction, and thus phase-matching considerations [17], which means that the equations below could also describe a single two-level system coupled to a cavity [18]. We use the usual input-output formalism for a single-sided, fairly high-finesse cavity. The basic equations are then

$$\begin{aligned}\dot{\sigma}(t) &= -i\Delta(t)\sigma(t) - \gamma\sigma(t) + ig(t)E(t) \\ \dot{E}(t) &= ig(t)\sigma(t) - \kappa E(t) + \sqrt{2\kappa}E_{in}(t) \\ E_{out}(t) &= -E_{in}(t) + \sqrt{2\kappa}E(t).\end{aligned}\quad (1)$$

Thanks to the linearity of the dynamics,  $\sigma$  and  $E$  can be interpreted as the atomic polarization and cavity fields (in the semi-classical regime), but also as the probability amplitudes corresponding to a single atomic excitation in the ensemble and a single cavity photon respectively (in the quantum regime, which is our focus here) [2, 6, 14];  $E_{in}$  and  $E_{out}$  are the incoming and outgoing fields (photon wave functions);  $g(t)$  is the time-dependent light-matter coupling, which is proportional to the transition dipole matrix element between the ground and excited atomic states (and also to  $\sqrt{N}$ , where  $N$  is the total number of atoms);  $\kappa$  is the cavity decay rate;  $\gamma$  is the atomic decay rate;  $\Delta(t)$  is a time-dependent detuning which may arise in practice as a consequence of applying a time-dependent external field in order to control the dipole element and thus  $g(t)$ ;  $\gamma$  and  $\Delta(t)$  are imperfections that we will neglect at first to keep the discussion simple, but whose effect will be discussed later in the paper. The described system is formally equivalent to a Raman memory in a cavity, if the excited state is adiabatically

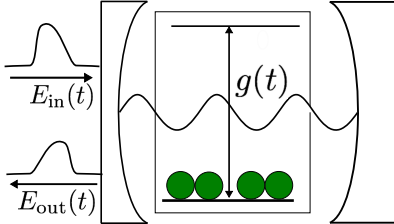


FIG. 1: We consider an ensemble of two-level systems inside a one-sided cavity, where the time-dependence of the light-matter coupling  $g(t)$  can be controlled. See also Eq. (1).

ically eliminated in the Raman case [14], and where the two-photon spin transition is replaced by a single-photon optical transition.

We are interested in the (realistic) situation where the cavity decay rate is by far the fastest relevant timescale. In this case it is well justified to adiabatically eliminate the cavity field, setting  $\dot{E} = 0$ . This gives

$$E(t) = \frac{1}{\kappa} \left( ig(t)\sigma(t) + \sqrt{2\kappa}E_{in}(t) \right) \quad (2)$$

and hence

$$\begin{aligned} \dot{\sigma}(t) &= -\frac{g^2(t)}{\kappa}\sigma(t) + i\sqrt{\frac{2}{\kappa}}g(t)E_{in}(t) \\ E_{out}(t) &= E_{in}(t) + i\sqrt{\frac{2}{\kappa}}g(t)\sigma(t) \end{aligned} \quad (3)$$

where we have set  $\Delta(t) = \gamma = 0$ , as mentioned above. It is straightforward to derive the (very intuitive) continuity equation

$$\frac{d}{dt}|\sigma(t)|^2 = |E_{in}(t)|^2 - |E_{out}(t)|^2. \quad (4)$$

We now discuss quantum memory operation, starting with a discussion of the read process. (The motivation for beginning with this will become clear in the following.) The read process corresponds to a situation where there is no incoming photon,  $E_{in} = 0$ . The continuity equation (4) implies

$$|\sigma(0)|^2 = |\sigma(t)|^2 + \int_0^t dt' |E_{out}(t')|^2, \quad (5)$$

which motivates the definition of the read efficiency  $\eta_r$  as

$$\eta_r = \frac{\int_0^\infty dt |E_{out}(t)|^2}{|\sigma(0)|^2}. \quad (6)$$

Here we have defined  $t = 0$  as the starting time of the read process.

The solution of Eq. (3) with  $E_{in} = 0$  is given by

$$\begin{aligned} \sigma(t) &= \sigma(0)e^{-\int_0^t dt' g^2(t')/\kappa} \\ E_{out}(t) &= i\sqrt{\frac{2}{\kappa}}g(t)\sigma(t). \end{aligned} \quad (7)$$

Using Eqs. (6) and (7) one finds

$$\eta_r = 1 - e^{-2\int_0^\infty dt g^2(t)/\kappa}. \quad (8)$$

Eq. (8) motivates the introduction of the effective time variable

$$\tau = \int_0^t dt' g^2(t')/\kappa, \quad (9)$$

see also Ref. [7], giving the simple expression  $\eta_r = 1 - e^{-2\tau_r}$ , where  $\tau_r = \int_0^\infty dt g^2(t)/\kappa$  is the total effective time

that elapses during the read process. This means that in order to maximize the read efficiency one simply has to maximize  $\tau_r$ . The shape of  $g(t)$  has an impact on the form of the output field, but the efficiency only depends on  $\tau_r$ .

In order to rewrite the whole dynamics in terms of the effective time variable  $\tau$ , we furthermore introduce effective input, output and cavity fields,

$$\mathcal{E} = \frac{\kappa}{g}E, \mathcal{E}_{in} = \frac{\sqrt{\kappa}}{g}E_{in}, \mathcal{E}_{out} = \frac{\sqrt{\kappa}}{g}E_{out}. \quad (10)$$

One then finds the new equations of motion (after adiabatic elimination of  $\mathcal{E}$ )

$$\begin{aligned} \frac{d}{d\tau}\sigma(\tau) &= -\sigma(\tau) + i\sqrt{2}\mathcal{E}_{in}(\tau) \\ \mathcal{E}_{out}(\tau) &= \mathcal{E}_{in}(\tau) + i\sqrt{2}\sigma(\tau). \end{aligned} \quad (11)$$

The read efficiency can be rewritten as

$$\eta_r = \frac{\int_0^{\tau_r} d\tau |\mathcal{E}_{out}(\tau)|^2}{|\sigma(0)|^2}. \quad (12)$$

The solution of Eq. (11) in the read case ( $\mathcal{E}_{in} = 0$ ) is simply

$$\sigma(\tau) = \sigma(0)e^{-\tau}, \mathcal{E}_{out}(\tau) = i\sqrt{2}\sigma(\tau). \quad (13)$$

This shows that in terms of the effective time (and of the effective fields) the read process is a simple exponential decay - a remarkable simplification considering that the time dependence of  $g(t)$  (and hence  $E_{out}(t)$ ) is completely arbitrary.

We are now ready to discuss the write process. We will immediately use the effective variables. Solving Eq. (11) for non-zero  $\mathcal{E}_{in}$  one finds

$$\sigma(0) = i\sqrt{2} \int_{-\tau_w}^0 d\tau' e^{\tau'} \mathcal{E}_{in}(\tau'), \quad (14)$$

where  $\tau_w$  is the total elapsed effective time for the write process and  $\sigma(-\tau_w) = 0$ . Note that no effective time elapses during times when the transition dipole is zero (i.e. during storage). We define the write efficiency as

$$\eta_w = \frac{|\sigma_f(0)|^2}{\int_{-\tau_w}^0 d\tau |\mathcal{E}_{in}(\tau)|^2}. \quad (15)$$

Our goal is to find the form of  $\mathcal{E}_{in}(\tau)$  that maximizes  $\eta_w$ . Since the solution for  $\sigma$  is linear in  $\mathcal{E}_{in}$ , maximizing  $\eta_w$  corresponds to maximizing  $|\sigma(\tau_w)|^2$  for a normalized input field satisfying  $\int_{-\tau_w}^0 d\tau |\mathcal{E}_{in}(\tau)|^2 = 1$ .

Before discussing the formal optimization, let us take a step back and try to make a guess for the optimum input field. We have seen that when expressed in terms of effective time rather than real time, the read process simply corresponded to exponential decay. It is natural

to suspect that inverting this decay (in effective time) will give the optimum effective input field. This means that our guess for the optimum solution is  $\mathcal{E}_{in}(\tau) \propto e^\tau$ .

This can be proved by functional differentiation. The optimum solution has to satisfy

$$\frac{\delta}{\delta \mathcal{E}_{in}^*(\tau)} \left[ |\sigma(0)|^2 + \lambda \left( \int_{-\tau_w}^0 d\tau |\mathcal{E}_{in}(\tau)|^2 - 1 \right) \right] = 0, \quad (16)$$

where  $\lambda$  is a Lagrange multiplier, and  $\mathcal{E}_{in}(\tau)$  and  $\mathcal{E}_{in}^*(\tau)$  are independent variables for each  $\tau$ . Solving this equation using Eq. (14) gives  $\mathcal{E}_{in}(\tau) \propto e^\tau$ , confirming the intuitive guess.

For this optimal solution the write efficiency is analogous to the read efficiency,

$$\eta_w = 1 - e^{-2\tau_w}. \quad (17)$$

The total efficiency (ignoring losses during storage) is then

$$\eta_{tot} = \eta_w \eta_r = (1 - e^{-2\tau_w})(1 - e^{-2\tau_r}), \quad (18)$$

which can obviously be simplified further if  $\tau_w = \tau_r$ . Provided that the optimum input field is chosen for the write process, the efficiency is thus maximized by maximizing  $\tau_w$  and  $\tau_r$ .

In real time the input field for the write process and the output field for the read process satisfy

$$\begin{aligned} E_{in}(t) &\propto g_w(t) e^{\int_{-\infty}^t dt' g_w^2(t')/\kappa} \\ E_{out}(t) &\propto g_r(t) e^{-\int_0^t dt' g_r^2(t')/\kappa}, \end{aligned} \quad (19)$$

where  $g_w(t)$  and  $g_r(t)$  are the light-matter coupling for the write and read processes respectively, and the proportionality constants are such that  $\int_{-\infty}^0 dt |E_{in}(t)|^2 = 1$  and  $\int_0^\infty dt |E_{out}(t)|^2 = \eta_{tot}$ . Eq. (19) shows that if the light-matter couplings are simple square functions in time, then the input and output fields are growing and declining exponentials in real time, respectively. However, there is no general requirement to choose the couplings in this way. On the one hand, one can achieve optimal write efficiency for any form of  $g_w$ , as long as the input field satisfies the above equation; on the other hand, the form of the output field can be tailored by choosing the form of  $g_r$ .

This means in particular that memory performance can be optimal even if the input and output fields are not related by time reversal in real time. For example, let us suppose that we want the input and output fields to have the same temporal shape,  $E_{out}(t) = -\sqrt{\eta_w \eta_r} E_{in}(t - T)$ , where  $T$  is the storage time, while still satisfying Eq. (19). By inverting Eq. (19) one can show that this can be achieved by choosing the following time-dependent couplings for the write and read processes:

$$g_w(t) = \sqrt{\frac{\kappa \eta_w |E_{in}(t)|^2}{2(1 - \eta_w + \eta_w \int_{-\infty}^t dt' |E_{in}(t')|^2)}}$$

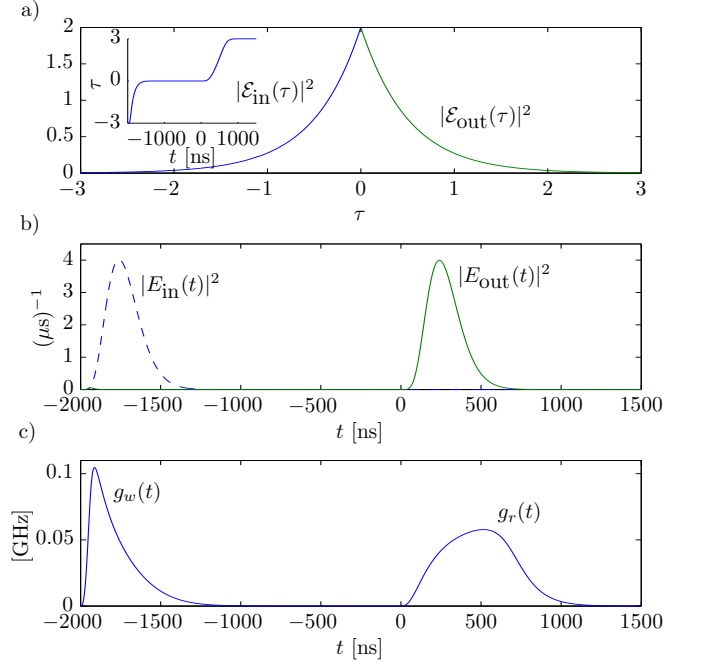


FIG. 2: Optimal memory performance is achieved when the effective input and output fields  $\mathcal{E}_{in}(\tau)$  and  $\mathcal{E}_{out}(\tau)$  of Eq. (10) are related by a reversal of the effective time  $\tau$  of Eq. (9) (a). This does not restrict the time dependence of the real fields  $E_{in}(t)$  and  $E_{out}(t)$  (b), provided the write and read couplings  $g_w(t)$  and  $g_r(t)$  are chosen appropriately (c). Any  $E_{in}(t)$  can be absorbed with the optimal efficiency  $\eta_w = 1 - e^{-2\tau_w}$  for  $g_w(t)$  satisfying Eq. (20); and  $E_{out}(t)$  can, for example, be chosen to be proportional to  $E_{in}(t - T)$  (where  $T$  is the storage time) for  $g_r(t)$  satisfying Eq. (20).

$$g_r(t) = \sqrt{\frac{\kappa \eta_r |E_{in}(t - T)|^2}{2(1 - \eta_r \int_{-\infty}^t dt' |E_{in}(t' - T)|^2)}}. \quad (20)$$

This choice of  $g_w(t)$  achieves the optimal write efficiency  $\eta_w = 1 - e^{-2\tau_w}$  for any input field  $E_{in}(t)$  and any value of  $\tau_w = \int_{-\infty}^0 dt g_w^2(t)/\kappa$ . On the other hand, the above choice of  $g_r(t)$  ensures that the output field is proportional to the input field (shifted in time by  $T$ ). We have seen that the read efficiency always satisfies  $\eta_r = 1 - e^{-2\tau_r}$  with  $\tau_r = \int_0^\infty dt g_r^2(t)/\kappa$ . Note that arbitrary output field shapes are possible for appropriately chosen  $g_r(t)$ .

So far we have neglected the spontaneous decay rate  $\gamma$ . It is not difficult to include in the above approach, but it obviously leads to somewhat lower efficiencies, because its effect is irreversible. The optimum input field can still be found by functional differentiation. To discuss the simplest example, let us consider square coupling pulses of strength  $g_{w(r)}$  and duration  $t_{w(r)}$ . Then the optimized input field for writing satisfies  $E_{in}(t) \propto g_w e^{\frac{g_w^2 t}{\kappa} + \gamma t}$  and the output field from the read process fulfills  $E_{out}(t) \propto$

$g_r e^{-\frac{g_r^2 t}{\kappa} - \gamma t}$ , while the efficiencies satisfy

$$\eta_{w(r)} = \frac{\frac{g_{w(r)}^2}{\kappa}}{\frac{g_{w(r)}^2}{\kappa} + \gamma} \left( 1 - e^{-2 \left( \frac{g_{w(r)}^2}{\kappa} + \gamma \right) t_{w(r)}} \right). \quad (21)$$

One can see that for large effective times the efficiencies tend towards  $\frac{C}{C+1}$ , where  $C = \frac{g^2}{\kappa\gamma}$ , which is essentially the optical depth in the presence of the cavity. High efficiencies require large  $C$ . For a given decay rate,  $C$  can in principle always be increased by increasing  $g$  (which requires increasing the dipole moment or the number of atoms), or by decreasing  $\kappa$  (which requires increasing the finesse of the cavity, i.e. the number of roundtrips).

The general case also includes a time-dependent detuning  $\Delta(t)$ . By functional differentiation one finds that the optimum input field has a phase dependence that exactly compensates the detuning. If this is not possible, the achievable efficiencies will again be reduced. However, in analogy to the case of spontaneous decay, the effect will be small as long as the ratio  $\frac{g^2}{\kappa\Delta}$  is large.

We will now discuss potential experimental implementations of the proposed protocol. In certain rare-earth ion doped crystals optical transitions can be switched on and off by changing the applied magnetic field [11, 12]. This is due to the coupling of the electronic Zeeman and hyperfine interactions in the presence of the crystal field. This coupling yields a substantial contribution to the overall nuclear Zeeman effect which is different for the ground and excited states, allowing one to control the branching ratios of optical transitions. For example, in Tm:YAG adding a field of order 80 mT transversally to a static applied field of 1 T will turn on a previously forbidden transition to a point where its optical depth  $d$  is of order 1/cm [11]. It is possible to control magnetic fields of this order (tens of mT) on ns timescales [19], making it possible to store light pulses whose duration is on these timescales. In practice the spectral width of the pulses is more likely to be limited by nearby transitions. The optical depth will be enhanced by the cavity, one has  $C \approx dF$  for the ratio  $C$  defined above, where  $F$  is the cavity finesse. Based on Eq. (21) high efficiencies should thus be achievable combining crystals of typical dimensions (say 1 cm in length) with moderate-finesse cavities.

The described memory could be attractive from a practical point of view as a solid-state Raman-like memory that does not require an optical control field, thus avoiding optical noise. Implementations in systems other than rare-earth ion doped crystals may be possible, for example using electric control fields for NV centers in diamond [13]. We have focused on the case of a memory inside a cavity. High efficiencies are possible without a cavity as well (for sufficient optical depth). The optimization of the input field is more complicated in this case and remains as work for the future. More conceptually, we

think that the present protocol has the potential to provide insight into the basic principles underlying quantum memories for light in general. As a first example, we have seen that the optimal write process is related to the read process by a reversal of effective, but not necessarily real, time. It is an interesting question whether the same holds for other memory protocols (beyond the obvious application to off-resonant Raman memories) for appropriately defined effective variables. See Refs. [6, 20–22] for related discussions in real time. Even more generally, the present protocol seems well placed to serve as an “archetype” for quantum memories, because, as discussed above, in all memory protocols the light-matter interaction is controlled in some fashion. Mapping various protocols onto the controlled-dipole memory discussed here may be a good way of analyzing their similarities and differences.

We thank Daniel Oblak for useful discussions. This work was supported by AITF, NSERC, the China Scholarship Council, General Dynamics Canada, and iCORE (now part of Alberta Innovates).

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